Estimating Power Losses due to Harmonics in Power Distribution System Components

Payam Tavakoli[1], Ali Palizban[2] and Constantin Pitis[3,*]

[1]*Electrical Engineering British Columbia Institute of Technology Burnaby, Canada*

[2]*Electrical Engineering British Columbia Institute of Technology Burnaby, Canada*

[3]*Director ELEN-MECH Consulting Inc. Vancouver Canada*

Abstract: This paper addresses the urgent need for accurate estimation of power losses caused by harmonics in various components of power distribution systems (PDS). Current methodologies for evaluating such losses in electrical distribution grids (EDG) are underdeveloped, highlighting the necessity for more refined analytical approaches. To address this gap, a mathematical model was developed to quantify harmonic-induced losses across key PDS components, with a focus on dry-type distribution transformers (DDTs). Expected accuracy range for estimating methodology was Class 2 accuracy range according to AACE[1]. Mathematical model was validated by using pre-existent tests performed on a 250 kVA DDT.

Power losses of DDTs are especially significant because these components play a critical role in power system efficiency and revenue generation. In the commercial and industrial sectors, approximately 50% of electricity passes[2] through DDTs, of which 40% of DDT efficiency is affected by harmonic losses [2][3]. As electrification and electric vehicle adoption grow, these losses are expected to increase. By identifying components most affected by harmonics, the model enables targeted mitigation strategies, potentially saving 600 GWh annually and reducing power losses by 70 MW[4] [3].

The paper concludes with a guide to the use of the proposed mathematical model for the estimation of Harmonic Power Losses and Applicable PDS Components (Appendix C).

Proposed mathematical model offers consultants, engineers and end-users a practical tool to improve transformer sizing, implement energy-saving measures, and enhance the efficiency and reliability of PDS. The outcomes support stakeholders, including utilities and customers, by reducing operational costs, extending equipment lifespans, and improving energy efficiency. The findings are a useful tool for government organizations and utilities in refining their energy efficiency programs and standards.

Keywords: Energy, Engineering, Harmonics, Motors, Power quality, power losses, Transformers.

1. INTRODUCTION

1.1. Power Quality in Electrical Distribution Grids

The increasing proliferation of nonlinear loads and generators used in smart grids and in the new distributed energy resources (DERs) interconnected with utility power systems and/or other distribution networks[5] introduces significant power quality (PQ) issues that need to be evaluated in new designs or existent PDS [4]. These networks are increasingly exposed to high harmonic content, primarily due to the widespread integration of non-linear loads without prior consultation or thorough power analysis. Modern power network designs must account for the estimation of harmonic distortion magnitudes. Furthermore, power plants incorporating renewable energy sources introduce additional complexities, necessitating meticulous management of power quality (PQ) parameters to ensure grid stability and efficiency.

Electric power quality is the degree to which the voltage, frequency, and waveform of a power supply system conform to established specifications. Deviations from these parameters lead to disturbances, with harmonics being among the most common issues.

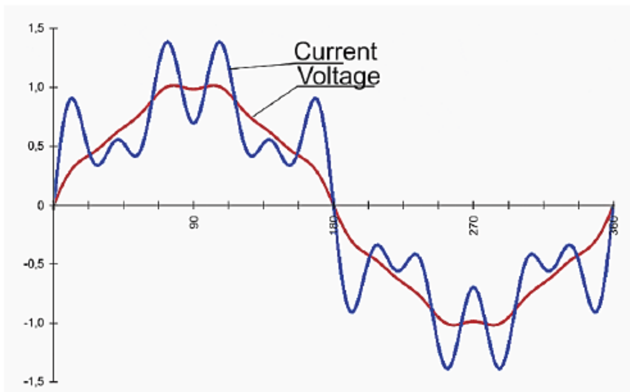

Figure 1: Network current and voltage superimposed with the 5th harmonic (5%), the 7th harmonic (4%) and the 11th harmonic (2.5%) [5].

*Address correspondence to this author at the Director Elen-Mech Consulting Inc. Vancouver Canada; E-mail: cdpitis@novuscom.net

1 AACE International "Costs estimate classification system as applied in Engineering" Recommended Practice 18R-97 https://buyandsell.gc.ca/cds/public/2021/10/28/2305b22cf3ce2931a9ca532a8274cba4/appendix_e_-_annex_7.2_-_classification_system_cost_estimate_matrix.pdf

2 According to Statistics, Canadian electricity consumption in 2020 was 522.2 TWh [1].

3 Harmonic losses in DDTs are significant, with annual estimates in Canada reaching 249 MW (2,070 GWh) [3].

4 Assuming that 0.5 % of this pass-through energy can be saved as a result of implementing energy conservation measures triggered by the harmonic power loss assessment in PDS [3].

5 IEEE 1547-2018 "IEEE Standard for Interconnection and Interoperability of Distributed Energy Resources with Associated Electric Power Systems Interfaces" https://standards.ieee.org/ieee/1547/5915/

E-ISSN: 2311-8741/24

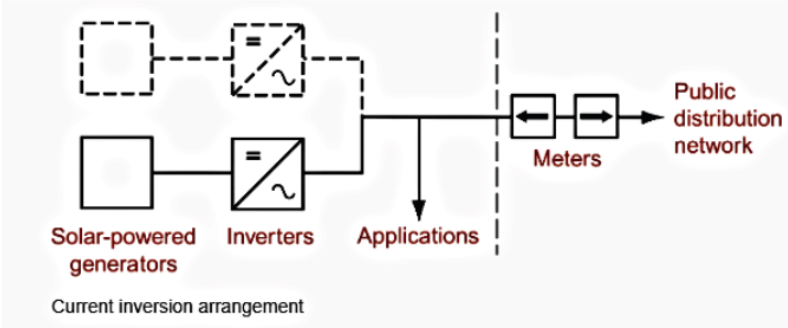

Figure 2: Proliferation of self-generation based on renewable energies feeding energy back into supply network may inject harmonics travelling towards lowest impedances found in the public distribution system (EEP – https://electrical-engineering-portal.com/less-known-phenomena-disturbance-electrical-installations).

Figure **1** illustrates fundamental wave distortion caused by the 5th, 7th, and 11th harmonics, which frequently occur in power distribution systems.

Harmonic generation occurs across various voltage levels and load types. Key sources include:

➢ **Residential Systems:** Low-voltage clusters are affected by household equipment such as TVs, PCs, UPSs, LED drivers, dimmers, and other electronics with voltage control technologies requiring inverters, converters, rectifiers.

➢ **Commercial and Industrial Systems:** The introduction of advanced motors, lighting, and switching technologies, along with devices like static converters, variable frequency drives (VFDs), and welding machines, contributes significantly to harmonics in low and medium voltage systems.

➢ **New Harmonic Generators:** Emerging sources include small power producers integrating distributed energy resources (*e.g.*, solar, wind, fuel cells) and distributed generation technologies (DGT) using electronic switching. These further increase harmonic distortion in grids.

➢ **Utility Systems:** Harmonics injected from power distribution systems can feed back into the utility network, especially in clusters of industrial facilities served by weak feeders, as shown in Figure **2** impacting power quality for other customers (function of their equivalent impedance)

Harmonics are distortions in voltage and current, often driven by nonlinear loads within the network[6]. The impact of harmonics generated by renewable energy sources (RES) and modern power electronic equipment, designed to improve energy efficiency, is not yet fully understood. While IEEE 519 regulates power quality at the point of common coupling (PCC), it lacks guidelines for mitigating harmonics' impact on customer-side equipment. Furthermore, IEC 61000-3-12, “Harmonic Measurements up to 75 A” sets limits for harmonic currents up to 75A but does not address broader losses.

Harmonic currents generated by nonlinear loads lead to several adverse effects on power distribution system components, including [4]:

- Excessive heat generation in equipment and cables, leading to accelerated aging and potential failures of motors, transformers and/or cables.
- Decrease in system capacity and efficiency, resulting in higher operational costs and reduced power delivery efficiency.
- Malfunction of protection and measurement devices, compromising system reliability and safety.
- Lowering of power factor – as shown in Appendix A.
- Reduced reliability and life expectancy of equipments
- Reduction of overall power distribution system efficiency.
- Increased operational costs.
- Permanent or temporary Resonance[7].
- Disturbances affecting sensitive loads.

[6] Power quality problems make their effects felt in three general areas:

- Power and energy costs.
- Downtime production.
- Equipment problems.

[7] The simultaneous use of capacitive and inductive devices in distribution networks may result in parallel or series resonance. The origin of the resonance is the very high or very low impedance values at the busbar level, at different frequencies. The variations in impedance modify the current and voltage in the distribution network.

1.2. On Deformant (Distortion) Power in Distribution Grids

Electrical power quantities are well-defined under sinusoidal conditions, but harmonics introduce complexities requiring additional considerations. Harmonics generate a power vector known as Deformant Power, first proposed by Prof. Ion Budeanu in 1955 [6]. These distortions, prevalent in non-linear loads, lead to significant power losses often overlooked in utility billing and affect revenue meter accuracy.

In circuits with harmonics, apparent power is composed of three components: Active Power (P), which performs useful work; Reactive Power (Q), which supports magnetic field creation; and Deformant Power (D), resulting from harmonic currents. This relationship is illustrated in Figure **3**. In case of non-sinusoidal signals, the inequality $S^2 > P^2 + Q^2$ is valid. The apparent power equation can be summarized as:

$$S^2 = P^2 + Q^2 + D^2 \quad (1)$$

Where the influence of Distortion power occurs. It consists of inequal harmonics of voltage and current

$$D = \sqrt{\sum_{j \neq k}^{\infty} \left[U_k^2 I_j^2 + U_j^2 I_k^2 - 2U_k I_k U_j I_j \cos(\varphi_k - \varphi_j)\right]} \quad [\mathrm{VA}] \quad (2)$$

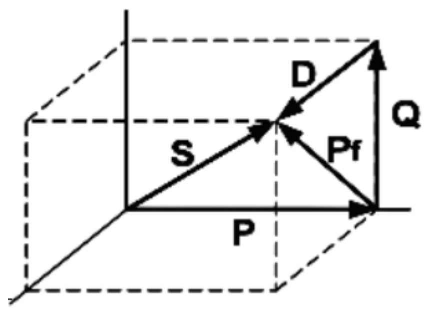

Figure 3: Power Triangle illustrating Distorted Power[8].

1.3. Limitations of Current Standards and Practices

Active power losses due to harmonic currents have been scarcely studied just for specific cases or configurations [7-9], but not for the whole range of power distribution system (PDS) components. IEEE 519[9] sets harmonic levels at the point of common coupling (PCC), requiring customers to manage harmonics to avoid utility penalties. However, it does not address harmonics within customer networks, leading to neglected harmonic levels and inefficiencies in PDS components like cables, motors, transformers, capacitors, UPSs, and control devices. This oversight underscores the need for a broader evaluation of harmonic-induced losses across PDS components.

Furthermore, Harmonic Mitigation Devices (HMDs), such as filters, are commonly used to address power quality issues but face significant challenges. These include high costs, increased system complexity, potential resonance conditions with passive filters, and limited bandwidth, which reduces their overall efficiency. Additionally, HMDs redistribute rather than eliminate harmonic-induced power losses. Due to their low impedances, these devices often absorb a substantial portion of harmonic-induced power losses, redistributing inefficiencies across the distribution grid and shifting power losses from other components[10].

1.4. Literature Review on Harmonic Losses

Research on harmonic impacts has largely focused on mitigation techniques and total harmonic distortion (THD), with limited exploration of power losses in specific PDS components. Studies have linked harmonic distortion in residential feeders to increased active power losses, proposing analytical methods to estimate these effects in urban distribution systems [10, 11]. A study on a 250 kVA single-phase transformer revealed that harmonics significantly elevate core losses, increase operating temperatures, and reduce efficiency and lifespan by up to 35% when supplying nonlinear loads [10]. Harmonics have been a longstanding concern, with early industrial electrification necessitating design considerations for arc furnaces and rectifiers in steel mills and electrochemical plants [11]. However, the advent of smart grids introduces new challenges, as the widespread use of power electronics at the consumer, generation, and grid levels injects higher harmonic currents, further exacerbating these issues [12]. While IEEE 519 provides general guidelines for harmonic management, it does not address the quantification of harmonic-induced power losses in critical components like transformers and motors[11].

---

[8] Kralikova E Cicakova O "Distortion Power Measurements" Measurement 2013, Proceedings of the 9th International Conference, Smolenice, Slovakia https://www.measurement.sk/M2013/doc/proceedings/075_Kralikova-1.pdf.

[9] IEEE 519 - Recommended Practice and Requirements for Harmonic Control in Electric Power Systems https://edisciplinas.usp.br/pluginfile.php/1589263/mod_resource/content/1/IEEE%20Std%20519-2014.pdf.

[10] Various harmonic mitigation devices (HMDs) are commonly installed in Motor Control Centers (MCCs) to address power quality issues. These low-impedance devices absorb harmonic power, converting it into heat, which increases the ambient temperature within the MCC. This rise in temperature often necessitates additional air conditioning systems, resulting in increased energy consumption to dissipate the heat generated by the HMDs. Notably, these devices lack standardized testing procedures for evaluating their temperature rise and power losses. Currently, no specific methodologies or test methods exist to assess these critical performance parameters.

[11] K factor transformers are specially constructed and grossly oversized (consequential yielding higher No-Load power losses) to handle the extra heat (power losses due to harmonics) and other stresses induced by harmonics, thus ensuring more reliable operation and longevity when serving non-linear loads.

This paper aims to address these gaps by developing mathematical models to quantify harmonic-induced losses in PDS components, equipping designers and end-users with tools to enhance efficiency, reliability, and economic performance.

2. EXECUTION METHODOLOGY

This paper will focus on creating mathematical models estimating power losses due to harmonics on conductors, cables, and electrical machinery (transformers and motors) subject of major power losses due to harmonics[12]. A comprehensive list of equations, along with the PDS components they are applicable to, is provided in Appendix D.

Traditionally, harmonic losses have been associated with harmonic currents, quantified using Total Harmonic Distortion of Current (THD_I). However, this paper also examines additional losses occurring in the magnetic circuits of electrical machinery, caused by harmonics generated by non-linear loads, as schematically illustrated in Figure **4**.

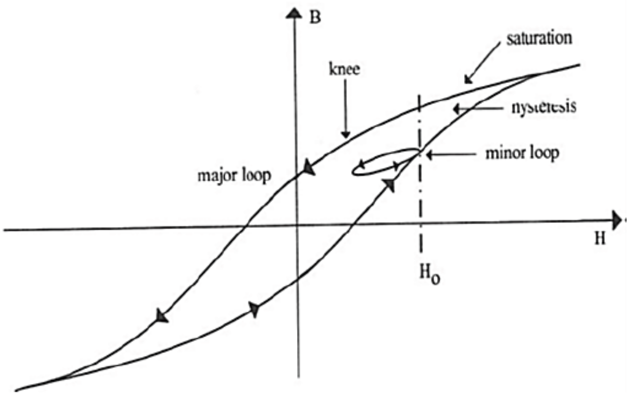

Figure 4: Schematics of voltage harmonics enabling harmonic transfer to higher voltage side of TRX [13].

3. POWER LOSSES DUE TO HARMONICS IN PDS COMPONENTS

3.1. Total RMS Current

The total RMS current (I_{RMS}) includes both the fundamental and harmonic components and can be expressed using the Total Harmonic Distortion (THD):

$$I_{RMS} = \boldsymbol{I_1\sqrt{1+THD^2}} \quad (3)$$

Where, I_1 is the RMS value of the fundamental (60 Hz) current. This relationship provides a foundation for estimating harmonic-induced power losses.

3.2. Power Losses in Three-Phase Systems

For three-phase systems, power losses can be calculated using the following formulas:

- Fundamental Power Loss at 60 Hz (P_1):

$$P_1 = 1.5 * R_{Line} * I^2_{1ine,true\ rms} \quad (4)$$

- Total RMS Power Loss, including Harmonics (P_{RMS}):

$$P_{rms}{}^{13} = 1.5 * R_{Line} * I^2_{Line,rms} \quad (5)$$

- Relationship Between Power losses at the Fundamental (60 Hz) and Total RMS values:

$$\frac{P_{rms}}{P1} = (1 + \mathrm{THD}_\mathrm{i}^2) \quad (6)$$

3.3. Cables & Conductors

For cases where phase currents and resistance values are known (*e.g.*, cables and windings), the total power loss at RMS values[14] can be expressed as:

$$P_{rms^{15}} = 3 * R_{phase} * I^2_{phase,rms} \quad (7)$$

These equations enable a practical assessment of harmonic-induced losses by measuring $I_{RMS}, I_1, and\ THD_i$. By leveraging established relationships, the study quantifies the additional losses caused by harmonics, providing insights for mitigating their impact on efficiency and reliability.

3.4. Impact of Skin Effect on R_{AC} and Harmonic Losses

Background research reveals limited studies evaluating power and energy losses due to harmonics in active equipment, particularly their impact on temperature rise. Harmonics exacerbate power losses through the skin effect, which increases AC resistance as current concentrates near the conductor surface at higher frequencies. This analysis improves upon traditional approaches, such as the 'Kskin' factor [14] and CSA C838-13 Annex B, which overlook frequency variation [15].

[12] Power electronic devices and capacitors will not be included in the scope of this project but can potentially be studied in future works.

[13] Applicable for both Delta and Wye configurations.

[14] Fundamental Power Loss at 60 Hz (P_1) are using true rms values (see formula (4).

The RMS value is needed to understand the actual usable energy. This value reflects the heating effect of the AC current, providing a clearer picture of the power that the system can deliver.

True RMS meters, measure both the peak value and a series of samples throughout the entire waveform. This results in a more accurate reflection of the waveform, especially in harmonic environments (distorted waveforms)

[15] For single-phase systems, the multiplier '3' in the equation should be disregarded.

AC resistance (R_{AC}) is calculated as:

$$Rac = \frac{l}{\sigma(2\pi a\delta - \pi\delta^2)} \quad (8)$$

where l is conductor length, σ is conductivity, a is radius, and δ is skin depth. Skin depth (δ) is given by:

$$\delta = \sqrt{\frac{\rho}{\pi\, f\, \mu_0 \mu_r}} \quad (9)$$

where ρ is resistivity, f is frequency, μ_0 is the permeability of free space, μ_r and is the conductor's relative magnetic permeability.

Validation against CSA C838-13 Annex B at 60 Hz [15] showed less than 3% deviation for cable sizes of 250–900 kcmil. The primary advantage of these equations is that, given a specific conductor material and size, R_{AC} can be calculated at harmonic frequencies, such as the 5th harmonic (*e.g.*, 300 Hz), allowing for more accurate assessments of harmonic-induced power losses in equipment such as motors, transformers, and reactors.

3.5. Sample Calculation of Power Losses in a Cable Feeding a Motor through a 6-Pulse VFD

To illustrate the application of these equations, consider a 300 kcmil copper cable, 100 feet in length, feeding a motor via a 6-pulse Variable Frequency Drive (VFD), with a harmonic current distribution characteristic of a 6-pulse VFD, as shown in Figure **5**.

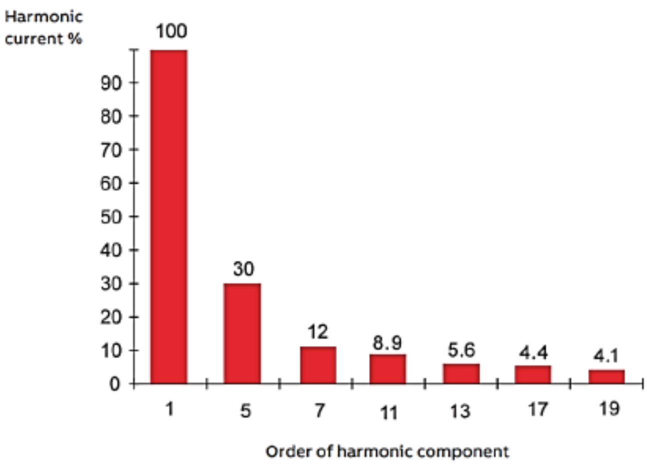

Figure 5: Harmonic Current Content of a typical 6-Pulse Rectifier [16].

Baseline Power Loss at Fundamental Frequency (60 Hz)

Using R_{AC} = 0.00318 Ω, calculated with Equations (8) and (9), the power loss at the fundamental frequency is:

$$P_{rms,fundemental} = 3 * 0.00318 * (240^2) = 0.55\ kW$$

Power Loss at the 5th Harmonic (300 Hz)

For the 5th harmonic, R_{AC} =0.00433 Ω, also calculated with Equations (8) and (9). Assuming the harmonic current is 30% of the fundamental:

$$P_{rms,5H} = 3 * 0.00433\ * (240\ x\ 0.3)^2 = 0.067\ kW$$

That is more than 12 % of the total power losses in the cable.

This example demonstrates how harmonics contribute to increased power losses in the cable, particularly at higher harmonic frequencies. By calculating losses at each harmonic, the total system losses due to harmonic effects can be accurately assessed.

4. POWER LOSSES IN INDUCTION MOTORS

The primary losses in an induction motor include stator core losses, stator winding losses, rotor winding losses, friction and windage losses, and stray load losses.

4.1. Harmonic Losses in Stator Winding

Stator winding losses are influenced by the difference between AC and DC resistances, particularly at harmonic frequencies where $R_{AC} > R_{DC}$ due to the skin effect. A practical method for calculating the harmonic power losses $P_{harm,losses,winding}$ in the stator windings could be obtained as difference between power losses due to entire effect of AC resistance (R_{AC}) and current (rms value) and fundamental power losses as follows:

$$P_{harm,losses,winding} = 1.5 * R_{AC} * I_{rms}^2 - 1.5 * R_{DC} * I_{1,line\ true\ rms}^2 \quad (10)$$

Where, The RMS value is needed to understand the actual usable energy. This value reflects the heating effect of the AC current, providing a clearer picture of the power that the system can deliver

I_{rms} = rms current
I_1 = fundamental current (“clean” wave)

Since the core laminations is made of conducting material, the EMFs[16] circulate currents within the body of the material as eddy currents. They will occur when the conductor experiences a changing magnetic field (fundamental and harmonics)[17].

[16]When the flux links with a closed circuit, an emf is induced in the circuit and the current flows, the value of the current depends upon the amount of emf around the circuit and the resistance of the circuit.

[17] The eddy current loss is minimized by making the core with thin laminations.

The equation of the eddy current loss is given as:

$$P_e = K_e B_m^2 t^2 f^2 V \quad (10.a)$$

Where,

- K_e – coefficient of eddy current. Its value depends upon the nature of magnetic material like volume and resistivity of core material, the thickness of laminations; usually adopted as $\pi^2/6\rho$ while considered that eddy current losses coefficient varies with frequency[18]
- B_m is maximum value of flux density in Wb/m^2
- t is thickness of lamination in meters
- f is frequency of reversal of the magnetic field in Hz
- V is the volume of magnetic material in m^3
- ρ is the material resistivity (usually $9.87*10^{-8}$ (Ohm.m)[19]

The depth of penetration (δ) graph is given by:

$$\delta = \frac{1}{\sqrt{\pi * f * \mu * \sigma}} \quad (10.b)$$

Where,

δ is the standard depth of penetration of the eddy currents in the material (m),

f is the excitation frequency (Hz), usually 300 Hs (considering the 5th harmonic having maximum weight)

μ is the magnetic permeability (H/m) of the material under test

σ is the electrical conductivity (S/m) of the material (usually $6.9*10^6$ S/m)

4.2. Losses in the Rotor Winding

Rotor winding losses are proportional to slip (s), which depends on the difference between synchronous and rotor speeds. The rotor power losses are calculated as:

$$Rotor\ Power\ Losses = Rotor\ Input\ Power * slip$$

Where Rotor Input Power is calculated as the difference between the Motor Input Power and the combined losses in the stator winding and stator core.

Rotor Power losses due to harmonics $P_{rotor\ losses,\ harmonics}$ can be estimated[20] as follows:

$$P_{rotor\ losses,\ harmonics} = \text{Rotor Input Power} * (\text{Actual slip} - Slip_{@60Hz}) \quad (11)$$

The actual slip S_{actual} > $Slip_{@60Hz)}$ because of harmonics generating parasitic torques in the motor

The $Slip_{@60Hz}$ was obtained by simulating working regimes squirrel cage induction rotor[21]

4.3. Core Loss (Due to Hysteresis)

Hysteresis loss occurs in the magnetic core of electrical machines, such as those found in transformers, electric motors, generators, and inductors. These cores are typically made from ferromagnetic materials like iron or iron-based alloys and are essential for guiding and amplifying magnetic flux. When these cores are subjected to an alternating magnetic field, they undergo repeated magnetization and demagnetization cycles. This continuous change in the magnetic state results in energy being dissipated as heat within the material, contributing to overall energy loss.

The specific power losses due to hysteresis in the motor stator core can be calculated as follows [24]:

$$p_h = k_h * f * (B_m)^n * V_C \quad (12)$$

Where k_h is the Steinmetz coefficient, f is frequency, B_m is maximum flux density, n is the Steinmetz index, and V_c is the stator core volume. For harmonics, these values are adjusted, *e.g.*, at the 5th harmonic (f=300 Hz), k_h and B_m are scaled accordingly.

Accurately assessing Friction and Windage losses, Stray Load Losses, and Stator Core Losses in electric motors often require detailed testing. When direct testing is not feasible, loss segregation techniques offer a practical alternative, particularly for squirrel cage induction motors. Data from the Electrical Apparatus Service Association (EASA) and the Association of Electrical and Mechanical Trades (AEMT) provide valuable estimates for rotor and stator winding losses[22], even when key parameters like slip are unavailable [17]. However, motor simulations or direct measurements remain essential for validation, highlighting an opportunity for future research to enhance these methods and improve accuracy.

5. POWER LOSSES IN TRANSFORMERS

Transformer losses can be broadly categorized into **no-load losses** and **load losses**:

---

[18] Popescu M: "On the Physical Basis of Power Losses in Laminated Steel and Minimum-Effort Modeling in an Industrial Design Environment IEEE-IAS Annual Meeting 2007 https://www.researchgate.net/publication/4280969_On_the_Physical_Basis_of_Power_Losses_in_Laminated_Steel_and_Minimum-Effort_Modeling_in_an_Industrial_Design_Environment

[19] https://etherndе.com/application/files/8016/2014/5733/Ether_NDE_ECT_Formulae_A2_Poster.pdf

[20] Neglecting the difference in friction and stray losses due to slip difference.

[21] Using Squirrel Cage Induction Motor Design SCDES2 – Wits University; software was used with Prof. Charles Landy permission.

[22] Tests performed on conditions of THD_I < 5 % on current harmonics.

No-Load Losses (Core Losses)

- **Magnetic Loading: Proportional to the voltage applied and representing losses when no external load is connected.**
  - **Hysteresis Losses:** Caused by repeated magnetization and demagnetization of the core material due to the alternating magnetic field.
  - **Core Eddy Current Losses:** Induced by circulating currents within the transformer core due to the changing magnetic field. These are minimized using laminated sheets of electrical steel.

Load Losses (Copper Losses)

- **Electrical Loading: Occurs when current flows through the windings as a load is applied.**
  - **I²R Losses:** The primary type of load loss, proportional to the square of the load current.
  - **Winding Eddy Current Losses:** Losses due to circulating currents induced in the winding conductors by leakage flux, significant at higher frequencies or with harmonics.
  - **Stray Losses:** Part of load losses, these include additional energy losses caused by leakage flux inducing eddy currents in components other than the windings, such as structural parts of the transformer (*e.g.*, tank walls, clamps). These losses are load-dependent and can be affected by harmonic distortion in the current.

5.1. Load Losses and Eddy Current Losses

Load losses include I^2R losses and winding eddy current losses, with stray losses typically negligible for dry-type transformers as per ANSI C57.110-1998 [18]. The total load losses can therefore be calculated as:

$$P_{Load,Total} = P_{I2R} + P_{ECR} \quad (13)$$

where P_{ECR} is the eddy current losses due to harmonics.

In dry-type transformers, eddy current losses, P_{ECR}, are typically estimated to be 15% of the *rated* I^2R loss (P_{LL-R}), as inferred from ANSI C57.110-1998. This is reflected in the typical per-unit value $P_{LL-R(pu)} = 1.15$, indicating that the rated winding current losses constitute 15% of the toral rated winding I^2R losses. Considering harmonic effects, the total winding load losses for a three-phase transformer are:

$$P_{Load,Xmfr} = 3 * I_1^2 R_1 + 3 * \sum_{h=2}^{Hmax} I_h^2 R_h + P_{ECR} \left(\frac{I_1}{I_R}\right)^2 + P_{ECR} \sum_{h=2}^{Hmax} \left(\frac{I_h}{I_R}\right)^2 h^2 \quad (14)$$

Where, according to [18],

R_1 is the fundamental phase resistance of the transformer in the windings,
I_1 is the fundamental (60 Hz) load current
R_h at each harmonic frequency h (R_h) in the winding,
I_h at each harmonic frequency h (I_h), the rated losses in the winding due to winding eddies (P_{ECR}),
h is the harmonic number (h),
I_R is the rated current

For single-phase transformers, the factor of 3 is removed. These formulas emphasize the significant impact of harmonic currents on transformer heating, as eddy current losses scale with the square of the harmonic order.

5.2. No-Load Losses (No-Load)

Core losses, also known as no-load losses, occur in the transformer core due to alternating magnetic flux and are independent of the load. These losses include hysteresis losses, caused by the repeated magnetization and demagnetization of the core material, and eddy current losses, induced by circulating currents within the core. While hysteresis losses depend on the core material properties and frequency, eddy current core losses scale with the square of the frequency and are minimized with laminated core designs. This analysis will focus solely on hysteresis losses, as they are typically the dominant component of core losses in properly laminated cores.

The specific harmonic power losses due to hysteresis in the transformer core can be calculated using the following equation:

$$p_{hys} = k_h * f * (B_m)^n * V_C \quad (15)$$

Where k_h is the Steinmetz coefficient, f is frequency, B_m is maximum flux density, n is the Steinmetz index, and V_c is the transformer core volume. Under the presence of harmonics, f and B_m are adjusted to account for higher frequencies and reduced flux density.

The no-load losses are obtained by direct measurements provided by manufacturer or repairer or by motor simulation[23]

[23] Using Squirrel Cage Induction Motor Design SCDES2 – Wits University; software was used with Prof. Charles Landy permission.

5.3. Total Transformer Losses

Under harmonic distortion, the total losses for a three-phase transformer are expressed as [18]:

$$P_{3phxmfr,h} = P_{hys,h} + 3 * I_1^2 R_1 + 3 * \sum_{h=2}^{Hmax} I_h^2 R_h + P_{ECR}\left(\frac{I_1}{I_R}\right)^2 + P_{ECR}\sum_{h=2}^{Hmax}\left(\frac{I_h}{I_R}\right)^2 h^2 \quad (16)$$

And for a single-phase transformer:

$$P_{1phxmfr,h} = P_{hys,h} + I_1^2 R_1 + \sum_{h=2}^{Hmax} I_h^2 R_h + P_{ECR}\left(\frac{I_1}{I_R}\right)^2 + P_{ECR}\sum_{h=2}^{Hmax}\left(\frac{I_h}{I_R}\right)^2 h^2 \quad (17)$$

The proposed equations will be applied using real data from a 250 kVA distribution transformer and validated against the results presented in R. Hasegawa's *"Energy Efficiency of Amorphous Metal-Based Transformers,"* which analyzed transformer losses under harmonic distortion[24] [19].

6. APPLYING MATHEMATICAL MODEL OF ESTIMATING POWER LOSSES DUE TO HARMONICS IN A DDT

6.1. Why Focus on DDTs?

Dry-Type Distribution Transformers (DDTs) are critical in power systems due to their environmental and safety benefits. However, harmonic currents significantly impact their performance, increasing core and winding losses, reducing efficiency, and accelerating aging. These engineering estimations address these challenges to provide insights into managing harmonic-induced losses.

The engineering estimations underscores the importance of this focus. Verhelst *et al*. proposed a derating method for DDTs based on current distortion parameters, highlighting the need to account for harmonics to avoid inefficiencies and failures [20]. Similarly, Digalovsky *et al*. demonstrated a direct link between high-order harmonics and elevated core losses in three-phase transformers, emphasizing the importance of targeted evaluations [21].

The engineering estimations aims to address these gaps and develop practical methodologies for assessing and mitigating harmonic-induced losses in DDTs.

6.2. Validation of Mathematical Model by using a 250 kVA Transformer

A single-phase 250 kVA transformer was used to validate the proposed equations against tests performed on a similar DDT type. Transformer specifications are detailed in Table **1**. This validation assesses the proposed methodology's accuracy in predicting harmonic-induced losses in DDTs and its applicability for practical scenarios.

The analysis incorporates real data from the transformer and compares results with R. Hasegawa's "Energy Efficiency of Amorphous Metal-Based Transformers," which evaluated losses under harmonic distortion [19]. The transformer was loaded at 58%, and its harmonic content spectrum is presented in Table **2**.

Obtaining accurate B-H characteristics, core dimensions, and winding arrangements for specific transformers is often impractical.

These engineering estimations provide a practical approach to estimating transformer losses under harmonic distortion using reasonable assumptions and approximations.

Key Assumptions

1. For hot-rolled silicon steel, a typical flux density (B) is 1.25 Wb/m² [22]. Multiplying by $\sqrt{2}$ to account for the RMS-to-peak relationship gives $B_m = 1.25 * \sqrt{2} = 1.7677\,T$.

2. The Steinmetz coefficient (k_h), specific to the core material, is computed using (9) from [27] for M400-5A (0.5 mm) silicon steel, (with hysteresis graph provided in Appendix C) commonly used in transformer laminations; Steinmetz coefficient variability can be taken in account by using Ewing equation [26} ensuring accurate loss estimation.

3. The transformer core volume (V_c)is estimated using $V_c = {}^{M}/_{\rho}$, where M is the core mass, and ρ the material density. Assuming a core-to-copper weight ratio of 3 to 1 and a total transformer weight of 808 kg, the core weight is 515 kg. With a steel density of $\rho \approx 7440\,kg/m^2$, V_c is calculated as $0.112\,m^3$.

4. A typical value of 2 will be used for the Steinmetz Index (n), as suggested in [24] and 27.

[24] The tests were performed on a 250 kVA single phase DDT at the Electrical Research and Development Association, ERDA, Mumbai, India.

5. The AC resistances R_1 and R_h are evaluated using Equations (8) and (9). Temperature corrections are applied to adjust copper resistivity.

6. The primary and secondary winding lengths are calculated based on the transformer's core dimensions and turns: $L_{Primary} = 46.1\ m, L_{Secondary} = 9.2\ m.$

7. Operating currents at 58% load are estimated as $I_1 = 241.28\ A$ (primary) and 1208.14 A (secondary). Harmonic currents (I_h) are scaled based on the harmonic spectrum provided by Hasegawa.

8. The eddy current losses P_{ECR} will be evaluated as 15% of the *rated* winding I²R losses, (or the current loading) consistent with the assumptions outlined earlier (ANSI C57.110-1998) [18].

9. It is considered that the harmonic effect is proportional to the square of the load with respect to the rated load value [28]

Table 1: Technical Specifications of 250 kVA Transformer

Catalog No.	SC250J-K/Z3
Product Type	Isolation Transformer
Phase	Single
Power Rating	250 kVA
Primary Voltage	600 V
Primary Max Current	416 A
Primary Terminal	H1-H2
Primary Terminal Lugs	2 of 300MCM, cross section 2x348 mm2, diameter 152 mm
Primary Conductor Cross Sectional Area	138 mm2
Primary Winding Turns	1075
Secondary Voltage	240/120 V
Secondary Max Current	2083.3A @ 120 V, 1041.7A @ 240 V
Secondary Terminal Lugs	X1-X4, Interconnect for Lower Voltage: X1 to X3 and X2 to X4, or for Higher Voltage: X2 to X3
Secondary Conductor Cross Sectional Area	694 mm2
Seconding Winding Turns	215
Winding Conductor	Copper
Average Temperature Rise, °C	150
Insulation Class	220 (R)
Weight	1110 kg

The Harmonic Impact on Transformer Losses complete testing data are presented in Appendix B

6.3. Analyzing the Results

The results of the transformer loss calculations under fundamental and harmonic distortion conditions are calculated based on Equation (17) and are summarized in Table **3**.

The calculated hysteresis, eddy current, and coil (winding) losses closely align with values reported in R. Hasegawa's study, demonstrating the accuracy of the proposed equations. Under harmonic distortion, total transformer losses were calculated at 2839.21 W, compared to 2524 W in Hasegawa's study, with a 12% deviation, which is within acceptable margins given the assumptions made with expected accuracy range Class 2 according to AACE[25]. The deviations are mostly due to power losses in the winding coils[26] (caused by difference in winding coils designs)

It is important to note that the transformer specifications in Hasegawa's study were not fully disclosed, necessitating assumptions about certain parameters, including core material properties, winding design and arrangements, and load conditions. Despite these uncertainties, the results demonstrate that the proposed methodology provides a robust framework for estimating transformer losses under harmonic distortion, validating its applicability for practical engineering scenarios.

Another critical point to note is that Hasegawa's study assumed that the hysteresis losses remained constant with varying harmonic distortion, meaning that the hysteresis losses of the core as a function of varying harmonic distortion were identical to those at the fundamental frequency.

This assumption relies on the idea that $B_{max}^n * f$ is constant, which implies that flux density decreases proportionally as frequency increases. This is consistent with the assumption that $V/_f$ (applied voltage to frequency ratio) remains constant, keeping the core unsaturated even with the presence of harmonics.

By contrast, the proposed methodology presented in this paper argues that harmonics inherently drive the transformer core closer to saturation, or even into a

[25] AACE International "Costs estimate classification system as applied in Engineering" Recommended Practice 18R-97 https://buyandsell.gc.ca/cds/public/2021/10/28/2305b22cf3ce2931a9ca532a8274cba4/appendix_e_-_annex_7.2_-_classification_system_cost_estimate_matrix.pdf.

[26] The Hasegawa study does not provide full name plate, winding and magnetic core details that should allow the correct identification of the DDT tested.

Table 2: Harmonic Content (THD ~25%)

Harmonics	1	3	5	7	9	11	13	15	17
Content (%)	100	1	20	10	1	9	6	1	5

Table 3: Transformer Losses Comparison Under Fundamental and Harmonic Distortion Conditions

Loss (w)	Fundamental	Including Harmonics (proposed Mathematical Model	Including Harmonics for Silicon Steel (Hasegawa)	% Difference (Harmonics)
Hysteresis	74.86	156.3	155.0	0.48%
Eddy Current	157.29	760.09	698.00	+ 8.89%
Coil Loss	1205.86	1923.37	1671.00	+ 13.10%
Total Transformer Loss	1438.00	2839.21	2524.00	+ 12%

saturated state. The presence of higher frequencies associated with harmonics increases the rate of flux change in the core, elevating the likelihood of core saturation.

Hasegawa's work assumes hysteresis losses remain constant under harmonic distortion; however, this analysis demonstrates that hysteresis losses increase with harmonic distortion. Since flux density (B_{ax}) is directly proportional to the applied voltage (V), the higher frequencies introduced by harmonics lead to an increase in B_{ax}, thereby resulting in greater hysteresis losses.

Moreover, while Hasegawa's study primarily focused on eddy current losses as the dominant contributor under harmonic distortion, it did not account for the substantial impact of hysteresis losses at higher frequencies.

The proposed mathematical model addresses this limitation by explicitly modeling the influence of harmonics on both the maximum flux density (B_{ax}) and frequency (f), enabling a more accurate estimation of hysteresis losses and total transformer losses under harmonic conditions.

7. CONCLUSION

These engineering estimations are devising a proposed mathematical model addresses the pressing need for accurate estimation of power losses caused by harmonics in power distribution systems (PDS), which are increasingly impacted by nonlinear loads and renewable energy sources. The estimations of the stator core losses are not using the "specific power losses" methodology for which full information of core losses ay different frequencies are required. Specific core losses (Watts/kg) provided by silicon steel manufacturers are the minimum values. During the processing of lamination cores the specific core losses increase due to specific technological process of stamping or laser cutting, stacking and core clamping with uncontrolled values. Core tests on rotating electrical machinery cannot provide real data on specific core losses due to fact that the only back iron is activated while stator core teeth are not.

Current methodologies for evaluating these losses remain underdeveloped, necessitating more refined approaches. To bridge this gap, a mathematical model was developed to quantify harmonic-induced losses across key PDS components, with a particular focus on dry-type distribution transformers (DDTs).

The validation of the proposed model by using a 250 kVA single-phase transformer being tested on specific harmonics spectrum demonstrated its accuracy and practical applicability. Under harmonic distortion, total transformer losses were calculated at 2839.21 W, with a 12% deviation compared to reference values. The deviation occurred mostly due to lack of information regarding the winding and core constructive details. This alignment underscores the robustness of the methodology in predicting harmonic-induced losses. The analysis further revealed the significant impact of harmonics on transformer cores, driving them closer to saturation. The increased flux density and frequency associated with harmonics amplify hysteresis losses, emphasizing the necessity for reliable loss estimation methodologies.

By identifying components most affected by harmonics, the proposed model enables targeted mitigation strategies, improving transformer sizing processes, reducing operational costs, and enhancing overall system efficiency.

A comprehensive guide to the use of the proposed mathematical model for the estimation of Harmonic Power Losses and Applicable PDS Components is presented in Appendix D.

APPENDIX A: TRUE (TOTAL) POWER FACTOR

The deterioration in power factor due to harmonics causes higher power losses affecting the performance of electrical machines and apparatus

The True (or total) Power Factor value that is affected by wave distortion is defined as follows:

$PF_{true} = PF_{displacement} \times PF_{distortion}$

$PF_{distortion} \approx 1 / \{\sqrt{[1 + (THD_V/100)^2]} \times \sqrt{[1 + (THD_I)^2]}\}$

The total harmonic distortion (THD) for voltage signals can be expressed as follows:

$$THD_V = \frac{\sqrt{V_2^2 + V_3^2 + V_2^4 +}}{V_1}$$

Where,

V_n = RMS value of the nth harmonic voltage
V_1 = RMS value of the fundamental voltage.

The formula can be further simplified to the following:

$$THD_V = \frac{\sqrt{\sum_{h=2}^{\infty} V_h^2}}{V1}$$

Following the same principles, the total THD for current signals can be expressed as follows:

$$THD_I = \frac{\sqrt{\sum_{h=2}^{\infty} I_h^2}}{I_1}$$

The Total Harmonic Distortion, which can be utilized to measure the harmonic distortion in voltage or current signals, is defined as the ratio of the root-mean-square of the harmonic content, considering harmonic components up to the 50th order and specifically excluding interharmonics, expressed as a percent of the fundamental (IEEE 519-2022).

For example, in a particular electrical grid, it was found that for a THDI of 70%, the True Power Factor was PF = 0.70, as shown in Figure **A.1**.

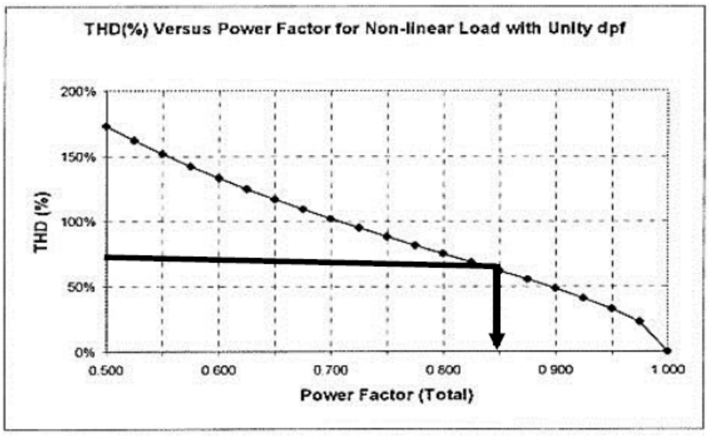

Figure A.1: The Influence of Current Harmonics on Power Factor.

Poor distortion power factor cannot be fully corrected by adding capacitors, which are typically used to improve displacement power factor. This is particularly critical in load areas dominated by single-phase power electronic loads, commonly found in commercial buildings, where distortion power factors are often very low. In such cases, adding compensating capacitors can worsen the situation by inducing resonances and increasing harmonic levels, further degrading the overall power factor.

APPENDIX B: HARMONIC IMPACT ON TRANSFORMER LOSSES – 250 KVA

Harmonic Content (THD~25%)

Harmonics	1	3	5	7	9	11	13	15	17
Content (%)	100	1	20	10	1	9	6	1	5

Transformer Losses without Harmonic Distortion

Loss (W)	Amorphous Metal	Silicon Steel
Hyteresis	99	155
Eddy Current	33	311
Total Core Loss	132	466
Coil Loss	966	1,084
Loading Level (%)	55	58
Total Transformer Loss	1,098	1,550

Transformer Losses with Harmonic Distortion of Table A

Loss (W)	Amorphous Metal	Silicon Steel
Hyteresis	99	155
Eddy Current	74	698
Total Core Loss	173	853
Coil Loss	1,553	1,671
Loading Level (%)	55	58
Total Transformer Loss	1,726	2,524

Source: Hasegawa, R. "Impact of amorphous metal-based transformers on efficiency and quality of electric power distribution" IEEE PES https://ieeexplore.ieee.org/abstract/document/970354/figures#figures

APPENDIX C: SAMPLE OF HARMONIC HYSTERESIS POWER LOSSES CALCULATIONS

For 250 kVA DDT the harmonic hysteresis power losses are obtained by using formula (15) in an excel document

nth harmonic	f	Bmax	n	Vc	Ph
1	60	1.767	2	0.112	75.11478
3	180	0.589	2	0.112	25.03826
5	300	0.3534	2	0.112	15.02296
7	420	0.252429	2	0.112	10.73068
9	540	0.196333	2	0.112	8.346087
11	660	0.160636	2	0.112	6.828616
13	780	0.135923	2	0.112	5.77806
15	900	0.1178	2	0.112	5.007652
17	1020	0.103941	2	0.112	4.418516
					156.2856

Total hysteresis losses (fundamental and harmonics) = 156.3 W.

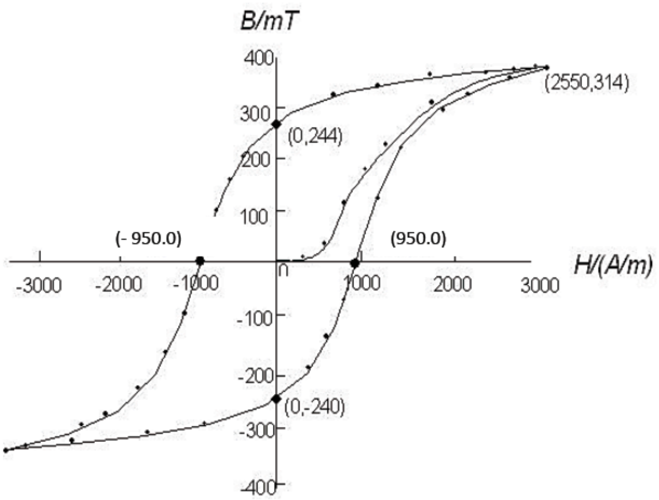

Figure C.1: Hysteresis graph for silicon steel used for evaluation of stator core losses.

APPENDIX D: GUIDE TO THE USE OF THE PROPOSED MATHEMATICAL MODEL FOR THE ESTIMATION OF HARMONIC POWER LOSSES AND APPLICABLE PDS COMPONENTS

Table D.1: Equations and Applicable PDS Components

Applicable PDS Components	Formula No.	Notes
Winding, Conductors (3-Phase Equipment)	(4), (5), (8)	Used for 3-phase equipment (regardless of their star or delta internal connection). The resistance and current values are LINE values and can be accounted for skin effect at harmonics by using Eq. (8).
Cables, Power Lines	(3), (7), (8)	Used when Phase values of the resistance and current are known (for example in the case of power lines and cables)
Total Transformer Loss (3-Phase)	(8), (9), (10), (12), (13), (15), (16)	For Dry-Type Transformers
Total Transformer Loss (Single-Phase)	(8), (17)	For Dry-Type Transformers
Squirrel Cage Induction Motors: For 70 % of losses (Stator, Rotor Winding & Stray)	(6), (7), (8), (9), (10), (11), (12)	Eq. (6) represents a general relationship between power losses (at fundamental 60 Hz and RMS values) and can be used to assess the increase in power losses in electric motors. This is achieved using the loss segregation chart proposed by EASA as a baseline. It is particularly useful when fundamental losses and THD% are known.
Squirrel Cage Induction Motors: Iron Core Loss	(6), (7), (8), (9), (10), (11), (12)	Eq. (7) focuses on core losses due to hysteresis. Total core losses, including eddy current losses and other potential factors, require further study to comprehensively evaluate their contributions under harmonic distortion. This represents an important area for future research to enhance modeling accuracy.

Proposed mathematical model offers consultants, engineers and end-users a practical tool to improve transformer sizing, implement energy-saving measures, and enhance the efficiency and reliability of PDS. The outcomes support stakeholders, including utilities and customers, by reducing operational costs, extending equipment lifespans, and improving energy efficiency. The findings are a useful tool for government organizations and utilities in refining their energy efficiency programs and standards.

CONFLICTS OF INTEREST

The author declared no conflicts of interest.

REFERENCES

[1] Index Mundi https://www.indexmundi.com/g/g.aspx?c=ca&v=81

[2] Pejovski D. et all "Impact of different harmonic loads on distribution transformers" Elsevier Procedia Engineering https://www.sciencedirect.com/science/article/pii/S1877705817342352

[3] Project Proposal Form CSA C802.5: 16 (R2020): Guideline for evaluating the efficiency of dry-type transformers under non-linear loading https://www.csagroup.org/store/product/C802.5-16/

[4] M. J. Ghorbani and H. Mokhtari, "Impact of harmonics on power quality and losses in power distribution systems," International Journal of Electrical and Computer Engineering (IJECE), vol. 5, no. 1, p. 166, Feb. 2015. https://doi.org/10.11591/ijece.v5i1.pp166-174

[5] E. Csanyi, "Practical design knowledge in harmonics distortion and Power Factor Correction (PFC)," EEP - Electrical Engineering Portal, https://electrical-engineering-portal.com/harmonics-distortion-power-factor-pfc-practical-design-knowledge#0

[6] D. Jeltsema, "Budeanu's concept of reactive and distortion power revisited," 2015 International School on Nonsinusoidal Currents and Compensation (ISNCC), pp. 1-6, Jun. 2015. https://doi.org/10.1109/ISNCC.2015.7174697

[7] B. -C. Neagu, G. Grigoras and F. Scarlatache, "Power losses estimation in harmonic polluted LV distribution networks with a fuzzy approach," 2016 8th International Conference on Electronics, Computers and Artificial Intelligence (ECAI), Ploiesti, Romania, 2016, pp. 1-6. https://doi.org/10.1109/ECAI.2016.7861106

[8] M. Chindris, A. Cziker, and A. Miron, "Software for computation of power in unbalanced and harmonic polluted industrial electric networks," in Saving energy--just do it!: ECEEE 2007 summer study: Conference proceedings, 2007

[9] B. C. Neagu, G. Grigoraş and F. Scarlatache, "The influence of harmonics on power losses in urban distribution networks," 2016 International Symposium on Fundamentals of Electrical Engineering (ISFEE), Bucharest, Romania, 2016, pp. 1-4. https://doi.org/10.1109/ISFEE.2016.7803184

[10] M. Shareghi, B. T. Phung, M. S. Naderi, T. R. Blackburn and E. Ambikairajah, "Effects of current and voltage harmonics on distribution transformer losses," 2012 IEEE International Conference on Condition Monitoring and Diagnosis, Bali, Indonesia, 2012, pp. 633-636. https://doi.org/10.1109/CMD.2012.6416225

[11] E. L. Owen, "A history of harmonics in power systems," in IEEE Industry Applications Magazine, vol. 4, no. 1, pp. 6-12, Jan.-Feb. 1998. https://doi.org/10.1109/2943.644881

[12] J. Luszcz, 'High Frequency Harmonics Emission in Smart Grids', Power Quality Issues. InTech, Apr. 17, 2013. https://doi.org/10.5772/52874

[13] Mohan G. Use of a Scaling Power Law to Incorporate Asymmetrical Minor Loops in the Inverse Jiles Atherton Model, IET Electric Power Applications, https://www.researchgate.net/publication/336326291_Use_of_a_Scaling_Power_Law_to_Incorporate_Asymmetrical_Minor_Loops_in_the_Inverse_Jiles_Atherton_Model

[14] CSA Group, Commercial Building Energy Data Reporting Best Practices and Requirements https://www.csagroup.org/wp-content/uploads/CSA-Group-Research-Commercial-Building-Energy-Data-Reporting-Best-Practices-and-Requirements.pdf

[15] CSA C838 – 13 (2022) Energy efficiency test methods for three-phase variable frequency drive systems https://www.csagroup.org/store/product/2421778/

[16] ABB, Technical Guide No. 6: Guide to Harmonics and Interharmonics in the Power System, 3AFE64292714 Rev F. Available: https://library.e.abb.com/public/bc35ffb4386c4c039e3a8ec20cef89c5/Technical_guide_No_6_3AFE64292714_RevF_EN.pdf

[17] Electrical Apparatus Service Association (EASA) and the Association of Electrical and Mechanical Trades (AEMT), "The Effect of Repair/Rewinding on Motor Efficiency – EASA/AEMT Rewind Study," https://www.theaemt.com/resource/revisited-the-effect-of-repairs-on-motor-efficiency.html#:~:text=The%20most%20recent%20study%2C%20conducted%20in%202019%2C%20by,unaffected%20by%20a%20repair%20using%20good%20practice%20procedures.

[18] ANSI, ANSI C57.110-1998: IEEE Recommended Practice for Establishing Transformer Capability When Supplying Nonsinusoidal Load Currents. American National Standards Institute, 1998.

[19] R. Hasegawa, Energy Efficiency of Amorphous Metal-Based Transformers, Metglas, Inc., Oct. 2004. Available: https://ieeexplore.ieee.org/abstract/document/970354/figures#figures

[20] Bart Verhelst *et al.,* "Derating method for dry-type power transformers based on current distortion parameters," EELAB/Lemcko, Ghent University – Belgium, Available: https://www.cired-repository.org/server/api/core/bitstreams/f1ec03c4-0cb1-449d-95bd-6cd7c3974e20/content

[21] M. Digalovsky et al., "Impact of current high order harmonic to core losses of three-phase distribution transformer," EUROCON 2013 IEEE, Available: https://www.researchgate.net/publication/261159122_Impact_of_current_high_order_harmonic_to_core_losses_of_three-phase_distribution_transformer https://doi.org/10.1109/EUROCON.2013.6625181

[22] Electrical Engineering XYZ, "Transformer Magnetic Flux Density," Available: https://www.electricalengineering.xyz/transformer-magnetic-flux-density/.

[23] J. Kartigeyan and M. Ramaswamy, "Effect of Steel Lamination on Core Losses in Switched Reluctance Motors," International Journal of Electrical Engineering & Technology (IJEET), vol. 7, no. 6, pp. 64-74, Nov.–Dec. 2016. [Online]. Available: http://iaeme.com/Home/issue/IJEET?Volume=7&Issue=6

[24] M. G. Say, Alternating Current Machines. New York: Wiley, 1978. https://openlibrary.org/books/OL3168643M/Alternating_current_machines

[25] IEEE Std: C57.110-1998 - IEEE Recommended Practice for Establishing Transformer Capability When Supplying Non-sinusoidal Load Currents https://ieeexplore.ieee.org/document/754767

[26] Steinmetz, C: "On the Law of Hysteresis" American Institute of Electrical Engineers Transactions, vol. 9, pp. 344,1892, http://edlab.wdfiles.com/local--files/pionering-papers/Steinmetz_1892.pdf

[27] Popescu M. *et al*. "Laminated Steel and Minimum-Effort Modelling in and Industrial Design Environment" IEEE - IAS 42nd Annual Meeting Oct 2007. https://www.researchgate.net/publication/4280969_On_the_Physical_Basis_of_Power_Losses_in_Laminated_Steel_and_Minimum-Effort_Modeling_in_an_Industrial_Design_Environment https://doi.org/10.1109/07IAS.2007.14

[28] Kanalik, A *et al*. "Calculation of Power Transformer Losses due to Harmonic Current Flow" The 10th International Scientific Symposium ELEKTROENERGETIKA 2019, 16-18. 9. 2019, Stará Lesná, Slovak Republic https://dusan.medved.website.tuke.sk/VEGA/VEGA-1-0372-18/clanky/Kanalik3.pdf

Received on 11-11-2024 Accepted on 15-12-2024 Published on 29-12-2024

https://doi.org/10.12974/2311-8741.2024.12.04